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Analysis and design of framed tube structures for tall concrete buildings

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During his 17 years of association with Skidmore, Owings & Merrill, Dr. Khan has been responsible for the engineering design of many major architectural projects. He has developed a number of new structural systems for tall buildings, both for reinforced concrete, as well as for structural steel. Among many of the significant tall structures designed by him, three special buildings that stand out are the 218 m (714 ft) tall One Shell Plaza building in Houston, which is the world's tallest reinforced concrete building, the 100-storey John Hancock Centre in Chicago, which is the world's tallest multi-use building, and the 110-storey Sears Tower in Chicago, now under construction, which will be the world's tallest building at 442 m (1450 ft) above ground. He has recently developed a new composite system for tall buildings, combining the advantage of both reinforced concrete and steel construction.
The 52-storey One Shell Square in New Orleans is being built using this system.
Dr. Khan is a member of the American Concrete Institute, the American Society of Civil Engineers, and the American Welding Society. He has published more than 50 technical papers relating to structural analysis and design, and has given lectures in many cities in Europe, the United States, Canada and Australia.
He is the founding Chairman of the Chicago Committee on High Rise Buildings.
Dr. Khan was cited three times as being among 'Men who served the best interests of construction industry' by Engineering News-Record *in 1966, 1968, and 1971. He received the Wason Medal for the most meritorious paper in* American Concrete Institute Journal *in 1971. He was awarded Chicagoan of the year in architecture and engineering by the Junior Chamber of Commerce. He was named Construction's man of the year by* Engineering News-Record *in 1972, and Chicago civil engineer of the year by the Illinois Section of American Society of Civil Engineers, 1972.*
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Synopsis:
The behaviour of framed tube structures is discussed from an overall structural system point of view.
The influence of various structural parameters is emphasized for achieving better tubular behaviour.
The concept of the equivalent reduced plane frame modelling technique is used for developing a series of 'influence' curves for the preliminary analysis and design. An example problem is worked out using these curves and the results are compared with a more exact solution by the computer.

Introduction

It is only in the last 20 years that reinforced concrete has been increasingly more used in the construction of tall buildings. Since the development of reinforced concrete construction in the early 1900s, the type of buildings using reinforced concrete was limited to only a few storeys high. The construction system used was the traditional beam-column frame system which made the construction of taller buildings relatively expensive and therefore economically unfeasible. In the early 1950s, shear wall type of construction was introduced. This immediately led to its use in apartment and office buildings which were as high as 30 storeys. Taller buildings still remained economically unattractive, because the shear walls mostly used in the core of the building were relatively small in dimensions compared with the height of such buildings, leading to excessive deflexion problems. It was obvious that the overall dimensions of the interior core were too small to provide economically the stability and stiffness for buildings over 30 or 40 storeys. The natural tendency then was to find new systems that would utilize the perimeter configurations of such buildings rather than the core configuration. The development of the framed tube system was therefore a logical outcome of this challenge. The framed tube system in its simplest form consists of closely spaced exterior columns tied at each floor with deep

spandrel beams, thereby creating the effect of a hollow concrete tube with perforated openings for the windows. Since the systems simulated a hollow tube using closely spaced columns in the perimeter frames, it is referred to as 'framed tube', Fig 1. This system was probably first applied on the design of the 43-storey DeWitt Chestnut Apartment Building in Chicago in 1963, Fig 2. Since then, the system has received wide acceptance among the designers all over the world, and many variations are being used in a number of buildings under construction.

From the point of view of construction economy, the framed tube compares favourably with the normal shear wall type of construction for medium rise buildings, but provides a distinctly economic advantage for taller buildings. Moreover, the closely spaced column system has the great advantage of also being the window wall system, thus replacing the vertical mullions for the support of the glass windows. In some recent buildings, the elimination of the traditional curtain wall with its metallic mullions was itself the justification for choosing this structural system.

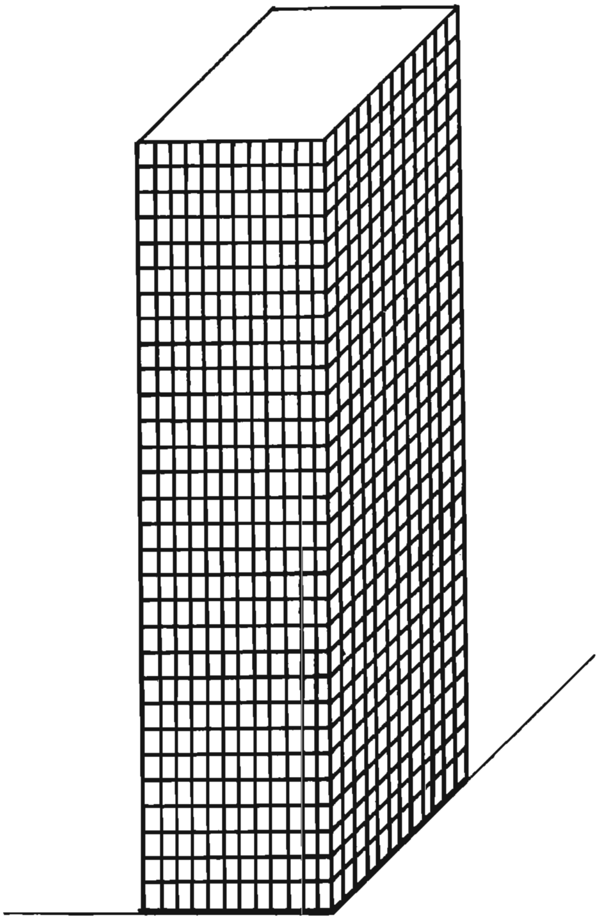


Fig 1. Framed tube

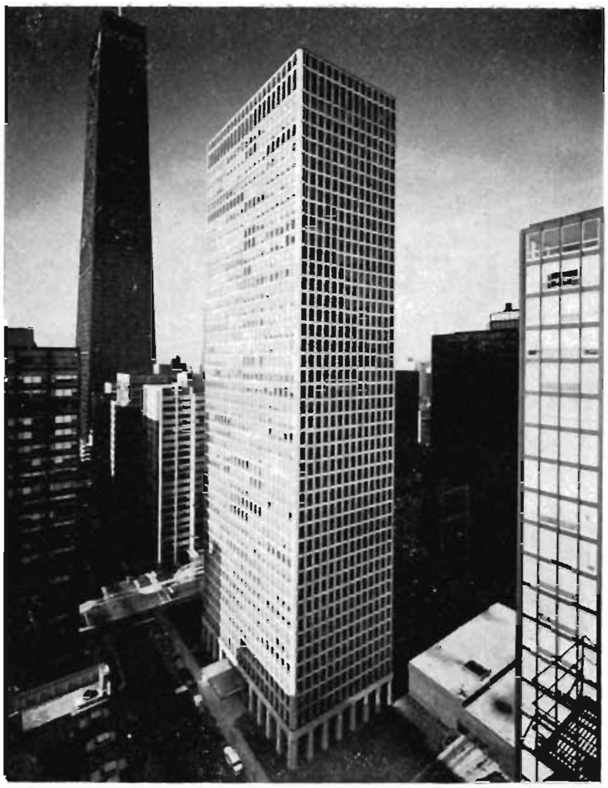


Fig 2. View of De Witt Chestnut Apartment Building

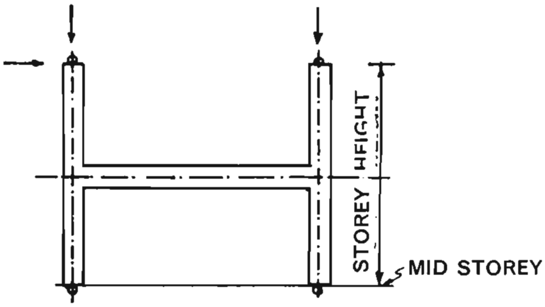


Fig 3. Model for optimum beam-column properties

In the framed tube structural system the exterior columns are generally spaced from 1·22 m (4 ft) centre to centre to a maximum of about 3 m (10 ft) centre to centre. Depending on the overall proportion and height of the building, the maximum spacing can probably be increased to 4·5 m (15 ft) centre to centre. The spandrel beams interconnecting the closely spaced columns generally vary from 600 mm (2 ft) in depth to about 1·22 m (4 ft) in depth with widths from 250 mm (10 in) to about 1 m (3 ft). In designing the framed tube structural system it is necessary to keep the proper balance of stiffness between the spandrel elements and the column elements so that neither of the elements presents inefficiency in terms of stiffness of the structure against lateral sway, or the overall strength of the tube system against lateral forces. In most recent structures, stiffness for limiting lateral sway controlled the proportions more often than the strength requirement. It is obviously advisable to optimize the subsystem represented by two columns and one spandrel, as shown in Fig 3, to arrive at the optimum spacing of the columns and the proportion of the spandrels and the columns in terms of overall architectural programme. In the DeWitt Chestnut Apartment Building, the columns were spaced at 1·65 m (5 ft 6 in) centres and the spandrels were 600 mm (2 ft) deep. The spacing of the columns in this case was also related to the module for interior planning of the apartment floors.

The framed tube structural system has expanded its application and its variations over the last five years. One of these variations is its application with an interior shear wall, commonly referred to as the 'tube in tube' system, as used in the 52-storey One Shell Plaza Building in Houston, reaching a height of 218 m (714 ft). Another variation of the framed tube system has been

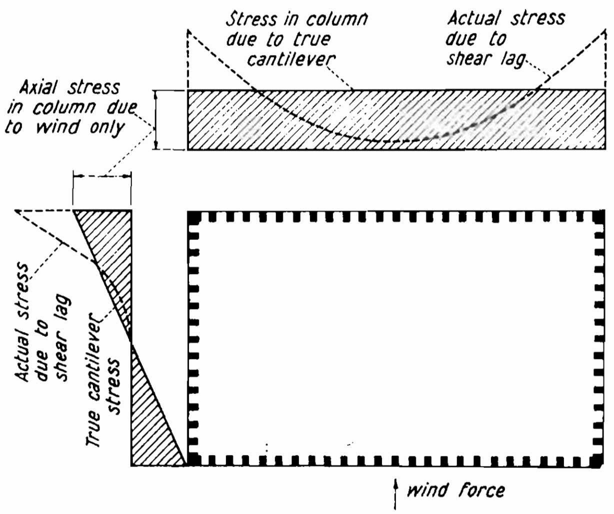


Fig 4. Shear lag in framed tube

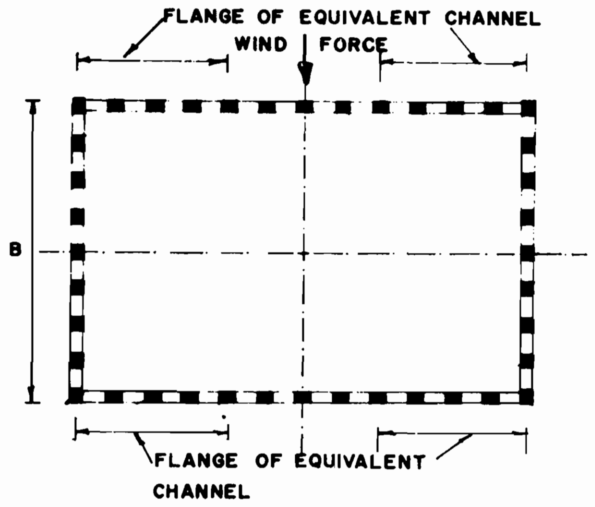


Fig 5. Equivalent channel for framed tube

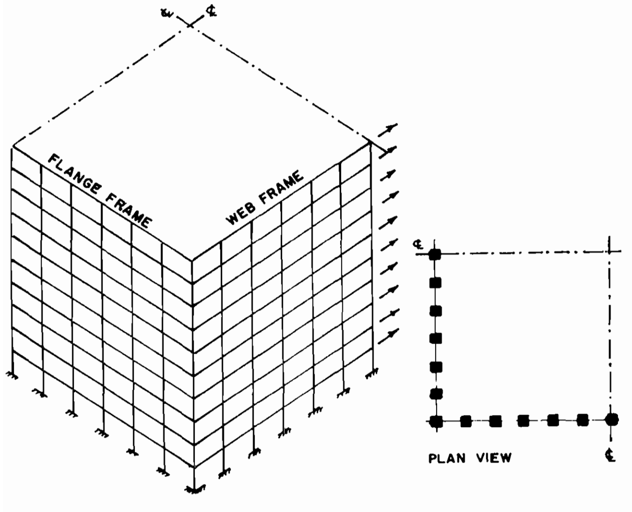


Fig 6. Space frame model

used in conjunction with a traditional steel frame for the interior of a building. This system, known as the SOM Composite System, has been used in three major tall buildings in the United States; namely, the 24-storey CDC Building in Houston, 35-storey Union Station Building in Chicago, and the 52-storey One Shell Square Building in New Orleans.

In view of the wide and varied application of the framed tube system, there is an obvious need for developing a preliminary analysis and design method for such a system. Of course, with the availability of large computer programs, the final solution may be easily refined. This paper is primarily intended for a clearer and better understanding of the behaviour of the framed tube system. A method of approximate analysis and designing for a wide range of proportion of the framed tube system is also presented.

Behaviour of framed tube system

The framed tube system behaves both as a true cantilever such as a shear wall as well as beam-column frame system. The overturning moment under the lateral load is resisted by the tube form causing compressions and tensions in the columns, whereas the shear from the lateral load is primarily resisted by bending in columns and beams on the two sides of the building parallel to the direction of the lateral load as shown in Fig 4. Therefore, for all practical purposes, the bending moments in these columns can be determined by judicious choice of the point of contraflexure in each storey. While it is true that in the lower few storeys, as well as in the upper few storeys, the point of contraflexure does not remain at the centre of the storey height, the intermediate storeys which constitute the major portion of the building generally have the point of contraflexure at mid-height of each storey. It is therefore possible to compute the bending moments in these columns within reasonable accuracy for any known lateral shear at each storey. One can, of course, make a simple iterative 'Maney-Goldberg' type slope-deflexion solution, or a modified moment distribution solution to determine more accurate moments in these columns. In fact, such an iterative solution will also give a good approximation of that portion of the total deflexion which is caused by the frame action only. To this, the additional overturning deflexion caused by tension and compression in the columns must be added to compute the total lateral deflexion.

The cantilever tube type behaviour becomes significant when the overturning of the entire building due to the lateral load is considered. For analysing the results of the overturning of the entire frame tube, the exterior column system can be considered as part of a rigidly diaphragmed hollow tube. However, because the webs of the hollow tube, that is the two sides parallel to the direction of the lateral force, are not truly rigid and solid webs, but are in fact beam-column frames, one must consider then the effect of loss of efficiency due to the flexibility of this 'web-frame' causing what is known as 'shear lag', as shown in Fig 4. For a very preliminary estimate of the overall resistance, as well as the deflexion of the building, the effective configuration of the 'tube' could be reduced to two equivalent channels resisting the total overturning moments, Fig 5. Experience has indicated that for very preliminary designs 'channel flanges' normally should not be more than half the depth of the web (walls parallel to lateral load), or more than about 10 per cent of the height of the building. These approximate rules have generally given maximum values of shear and moment, and the actual forces in the exterior

columns, are reasonably consistent with the exact theoretical analysis performed later for the final design by more generalized computer programs available today, such as STRESS, STRUDL, etc.

First preliminary design approach

The overturning moment resisted by the two channels will produce axial forces in the closely spaced columns in those channels, as well as shear forces in the connecting spandrels. The preliminary analysis of the axial forces in the columns as well as the shears in the connecting spandrel can be based on the classical beam theory and can be expressed as follows:

$$P_w = \frac{M \times C \times A_c}{I_e}$$

and

$$V_s = \frac{V_w \times Q \times h}{I_e}$$

where P_w = Axial force due to wind
M = Overturning moment
I_e = Effective moment of inertia of the tube
V_s = Spandrel shear
V_w = Total wind shear
Q = Sum of first moment of column areas about the neutral axis
C = Distance of any column from the neutral axis
h = Storey height
A_c = Cross-sectional area of column

From the structural point of view the framed tube is unfortunately full of too many large openings. Its behaviour is therefore of a hybrid nature showing characteristics of the pure frame as well as those of the pure tube. Even though one may expect the framed tube behaviour to be more like a tube resisting the lateral forces through axial forces in the columns, significant moments develop in the two walls parallel to the wind direction.

In a framed tube type structure the shears in the connecting beam as determined by the preliminary design method will generally indicate a reasonably uniform shear force in the spandrel beams along the two exterior walls parallel to the wind force. The resulting moments in the spandrel should, therefore, be used for preliminary design of these spandrels. The preliminary design of the closely spaced columns should be based on the known dead-load and live-load forces together with the axial force due to overturning in combination with the moments caused by storey shears.

Preliminary design using 'influence' curves

In order that framed tube structures of any aspect ratio and any height within practical range can be made more accurately than the equivalent channel method proposed earlier, the authors have developed 'influence' curves which can be directly used for making a relatively accurate preliminary design. These curves have been developed on the basis of a number of computer runs on ten-storey equivalent framed tubes with variable non-dimensional parameters representing ratios of shear stiffness S_b of the spandrel to the axial stiffness S_c of the columns, and a linearly varying ratio of bending stiffnesses of columns to spandrel. A study of the above ratio for a few framed tube buildings ranged from 0·95 at roof to 0·4 at ground. For this study, a ratio of 0·75 at roof to 0·5 at ground level is assumed. A separate computer run, with a constant ratio of 1·0, did not change the qualitative results.

The significant structural properties affecting the tube action are:

1. Bending stiffness:

$$K_c \text{ for column} = \frac{I_c}{H}$$

$$K_b \text{ for spandrel beam} = \frac{I_b}{L}$$

2. Shear stiffness of the spandrel beam:

$$S_b = \frac{12EI_b}{L^3}$$

3. Axial stiffness of the column:

$$S_c = \frac{A_c E}{H}$$

where, I_c = moment of inertia of the column
I_b = moment of inertia of the spandrel beam
A_c = cross-sectional area of the column
H = height of column
L = effective span of the spandrel beam
E = modulus of elasticity

the controlling parameters of framed tubes are:

$$\text{Stiffness ratio} = \frac{K_c}{K_b}$$

$$\text{Stiffness factor } S_f = \frac{S_b}{S_c}$$

$$\text{Aspect ratio } R = \frac{\text{flange frame}}{\text{web frame}}$$

The framed tubes have been analysed for uniform lateral load and for the following ranges of variables: aspect ratio values of 0·5, 0·666, 1·0, 1·5 and 2·0. Stiffness factor values of 0·1, 1·0, 10·0, and a linearly varying stiffness ratio of 0·75 at roof to 0·5 at ground level for all values of aspect ratio and stiffness factor combination.

Analysis of the equivalent framed tube from which the design curves have been developed was based on the configuration as shown in Fig 6. Direct solutions were obtained by converting the framed tubes into equivalent plane frames as shown in Fig 7.

Use of non-dimensional curves for preliminary design

A total of nine non-dimensional design curves, Figs 8 to 16, have been presented in this paper. These curves are primarily for computing column axial force coefficients for flange and web frame columns and shear force coefficients for the web frame beams. All these coefficients relate to unit values for the corresponding ideal tube forces. The main purpose of developing these curves was to provide the design engineer with a tool to determine the tubular characteristics of any given framed tube, and quickly to compute the total deflexion and bending moments and shears in the beams caused by the tubular nature of the entire structure. To make all the curves applicable to a wide range of realistic proportions of actual foreseeable buildings, these curves have been plotted against non-dimensional parameters representing the basic properties of the column and beam elements and aspect ratios.

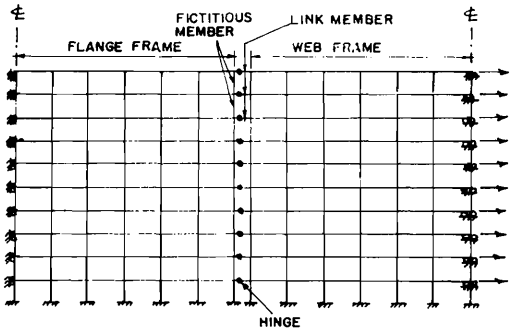


Fig 7. Equivalent plane frame

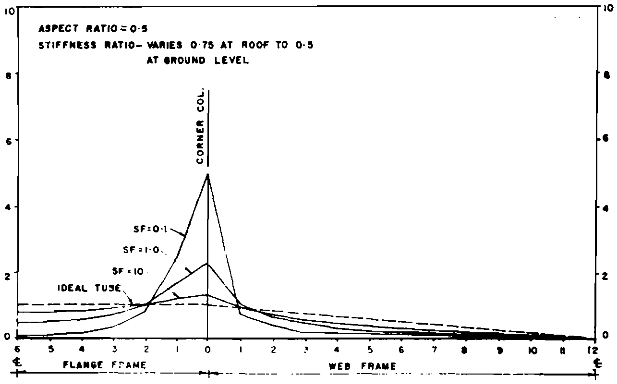


Fig 8. Column axial force coefficients, Level 1

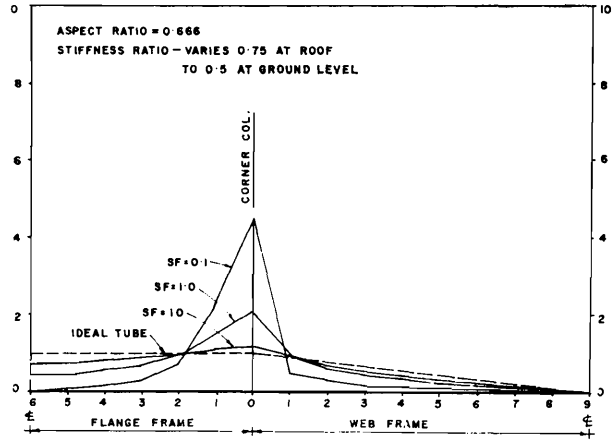


Fig 9. Column axial force coefficients, Level 1

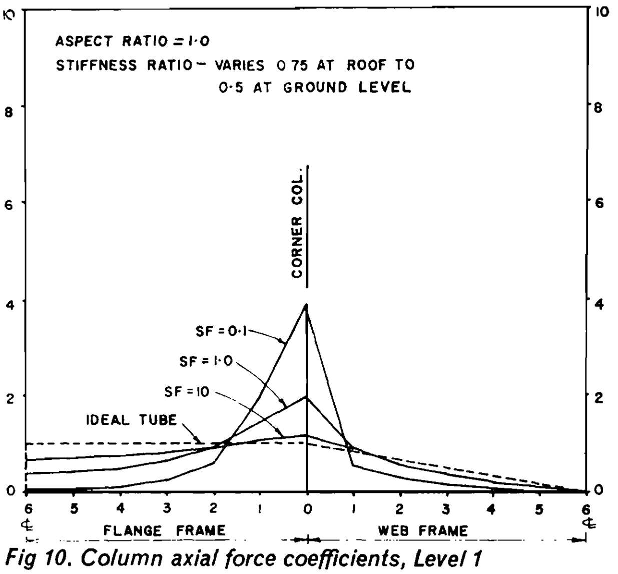


Fig 10. Column axial force coefficients, Level 1

To make the curves usable for any number of storeys as well as any number of columns around the perimeter of a building, reduction model techniques similar to those used by Khan in previous papers[1,2,3] have been used. A brief description of the modelling technique is described below.

Reduction modelling approach

In order to achieve uniformity of reference to the design curves, computer solutions were obtained on a number of specific ten-storey frame-tubes, each representing stiffness factor, aspect ratios and stiffness ratio indicated on the curves. Although the solutions were obtained on ten-storey hypothetical framed tubes, it can be shown that the tubular behaviour of any framed tube of any number of storeys can be simulated by converting it into an equivalent framed tube of the same height, but having less or more number of storeys. For any given plan proportion commonly referred to as the aspect ratio 'R' of a building, the variables that directly affect the tubular stress distribution are the shear stiffness S_b of the beams between columns and the axial stiffness S_c of the columns. For reasonably uniform spacing of perimeter columns S_b and S_c represents the sum total of all columns and beams around the perimeter. If, however, stiffness factor varies on each face, an average value can be taken for a reasonable approximation.

It can be shown that any framed tube of 'N' storeys can be reduced to a ten-storey equivalent tube of the same height by considering a transformation of the actual stiffness factor to a ten-storey equivalent stiffness factor of S'_{f10} where

$$S'_{f10} = S_f \times (N/10)^2$$

Using this simple reduction model technique, any given actual framed tube can first be converted to an equivalent ten-storey framed tube and then analysed by using the 'influence' curves.

Total behaviour of a framed tube

As already mentioned, the framed tube system always has two components of its behaviour: first, the frame action of the two sides parallel to the direction of the lateral load or movement and secondly, the overturning action of the entire tube causing only tension and compression in the exterior columns. All the 'influence' curves presented in this paper are for the evaluation of the tube action only, although the use of these curves will allow one to compute the approximate moments and shears in the spandrels which will define one boundary of the column moments. The other boundary of the column moments at any typical storey should be obtained by assuming the point of contraflexure at midheight of each storey. For preliminary design, the higher of the two values should be used.

To compute the total deflexion, the deflexion due to frame action must be calculated separately and added to the deflexion caused by tube action.

Example problem

To illustrate the use of 'influence' curves for preliminary designs, a hypothetical framed tube which is 50 storeys high and 40 × 55 m (130 × 180 ft) in plan and columns at 3 m (10 ft) on centre with 4 m (13 ft) floor to floor height is considered. Before using the 'influence' curves, the basic parameters of the structure, like aspect ratio, average stiffness factor, average stiffness ratio and frame deflexion, must be computed. Also, the deflexion and forces in column and beam for an ideal tube of the same aspect ratio must be known. Basic

TABLE 1
Computations of basic parameters for equivalent ten-storey model

Floor	Column properties			Spandrel beam properties			Stiffness factor $S_f = S_b/S_c$	$S_{f10} = S_f(N/10)^2$	Stiffness ratio $= K_c/K_b$
	Area In²	M. of I. In⁴	Axial stiffness S_cK/In	Area In²	M. of I. In⁴	Shear stiffness S_bK/In			
1— 5	972	462000	24400	1455	452000	13000	0·53	13·2	0·795
6—10	810	447000	20400	1200	372000	10700	0·525	13·1	0·925
11—15	810	447000	20400	1200	372000	10700	0·525	13·1	0·925
16—20	810	447000	20400	1200	372000	10700	0·525	13·1	0·925
21—25	717	400000	18000	1058	330000	9520	0·525	13·1	0·935
26—30	615	363000	15400	940	282000	8140	0·52	13·0	1·0
31—35	615	363000	15400	940	282000	8140	0·52	13·0	1·0
36—40	510	307000	12800	754	234000	6750	0·52	13·0	1·01
41—45	510	307000	12800	754	234000	6750	0·52	13·0	1·01
46—50	510	307000	12800	754	234000	6750	0·52	13·0	1·01

Average stiffness factor for ten-storey model = 13·05

Average stiffness ratio = 0·95 Aspect ratio $= \frac{180}{130} = 1{\cdot}4$

For this example use stiffness factor = 10·0
and aspect ratio = 1·5

TABLE 2
Computations for ideal tube

Floor	1	5	10	15	20	25	30	35	40	45
Overturning moment kip-ft	975000	789750	624000	477750	351000	243750	156000	87750	39000	9750
Tubular moment of inertia = ΣAR^2 ft⁴	333444	277870	277870	277870	245966	210975	210975	174955	174955	174955

$$\text{Column axial force at Level 1} = \frac{M \times C}{I} \times A_c = 1280^k$$

$$\text{Spandrel beam shear at centre line spandrel} = \frac{VQ}{I} \times H = 2850^k$$

section properties and parameters are presented in Tables 1 and 2.

Frame deflexion can be computed by any standard method and is approximately 94 mm (3·79 in). From Table 2, by conjugate beam method cantilever deflexion is computed as 190 mm (7·5 in). Any other similar method could also give reliable results.

The actual forces in columns, shears in beams and deflexion of the system, can now be computed using the 'influence' curves. For this example, Fig 11, $SF = 10$ curve is used for computing the column axial forces in first storey columns, and Fig 16 (c), $SF = 10$ curve is used for computing the beam shears in the first storey beams of a ten-storey model. The above results are summarized in Table 3 and compared with the results obtained from a more exact computer solution.

The deflexion of the total system is the sum of the frame deflexion plus the ideal tube deflexion times the magnification factor. This magnification factor is defined as the ratio of the sum of the ideal tube column forces to the actual column forces of the flange columns. For this example, the sum of the ideal tube column forces is 5·69 × 10 = 56·9 MN (1280 × 10 = 12800 kips) and the sum of the actual column forces is 480 MN (10825 kips). Hence, the magnification factor is 1·17.

Δ Total = 94 + 190 × 1·17 = 317 mm (3·79 + 7·5 × 1·17 = 12·49 in)

Δ Actual = 292 mm (11·51 in)

Discussion on results

Reviewing the above results in the use of 'influence' curves gives fairly reliable results and up to an accuracy of 90 to 95 per cent. The shear forces in web frame beams are apart, but near the centre of the frame, the

TABLE 3

Comparison of actual and calculated forces in columns and spandrel beams

Flange column axial forces			Shear force in web frame beams		
Col. location	Using influence curves kips	Actual kips	Beam location	Using influence curves kips	Actual kips
0	1660	1583	1	2340	1954
1	1530	1426	2	2620	2257
2	1280	1237	3	2750	2450
3	1150	1100	4	2810	2565
4	1020	1001	5	2850	2640
5	900	929	6	2850	2690
6	830	877	7	2850	2700
7	830	841			
8	830	818			
9	795	806			

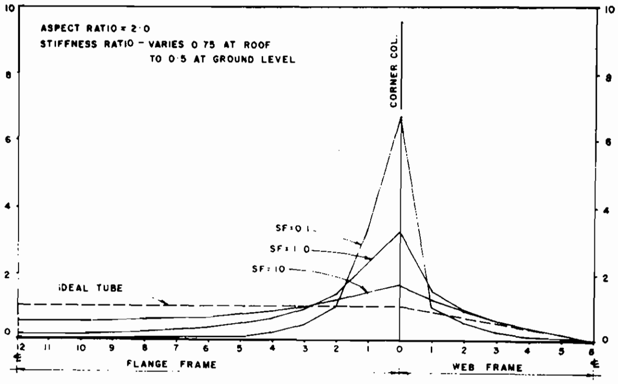


Fig 11. Column axial force coefficients, Level 1

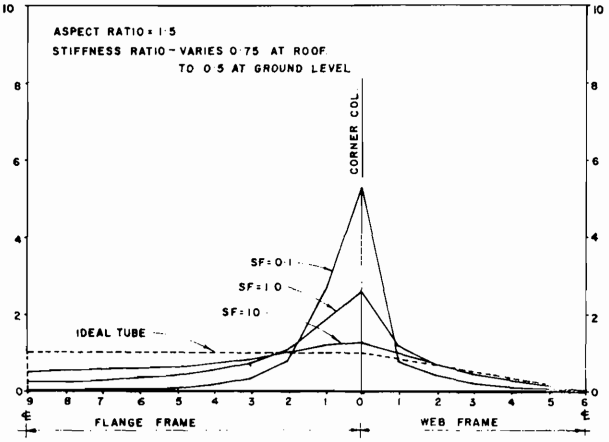


Fig 12. Column axial force coefficients, Level 1

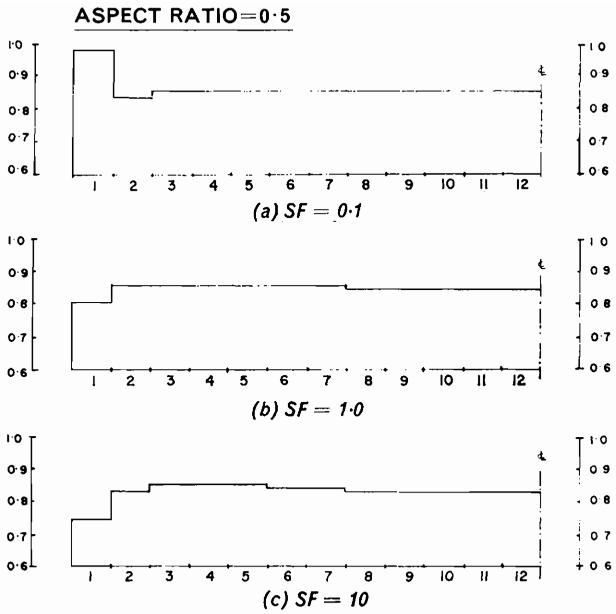


Fig 13. Shear force coefficients in web frame beams

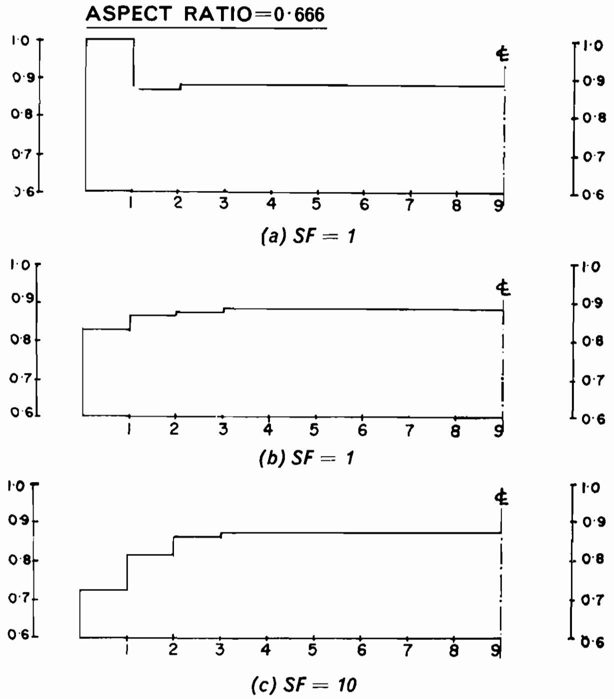


Fig 14. Shear force coefficients in web frame beams

forces are within 5 per cent. Since for a typical framed tube structure, the size of the spandrel beam is generally constant at a level, the inaccuracy in the corner spandrel beam shears is not of much consequence.

Conclusion

A framed tube type of structural system is increasingly being used for tall concrete as well as steel buildings. While large capacity computers can be used to finalize the design of such structures, they cannot substitute the improved understanding of such systems. Furthermore, the use of computers for preliminary design of even a moderate size framed tube may prove too expensive. Some simple preliminary design methods have therefore been described. It is hoped that the 'influence' curves which can be used for most practical

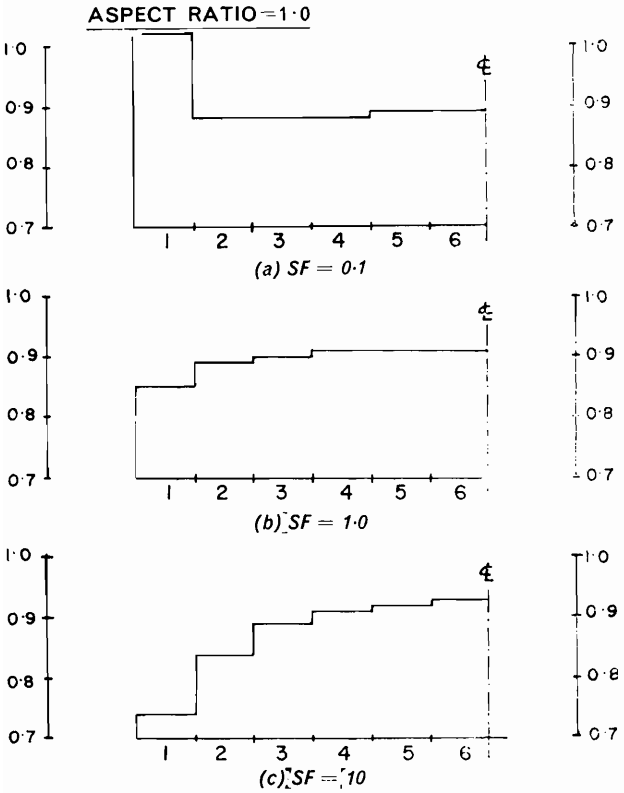
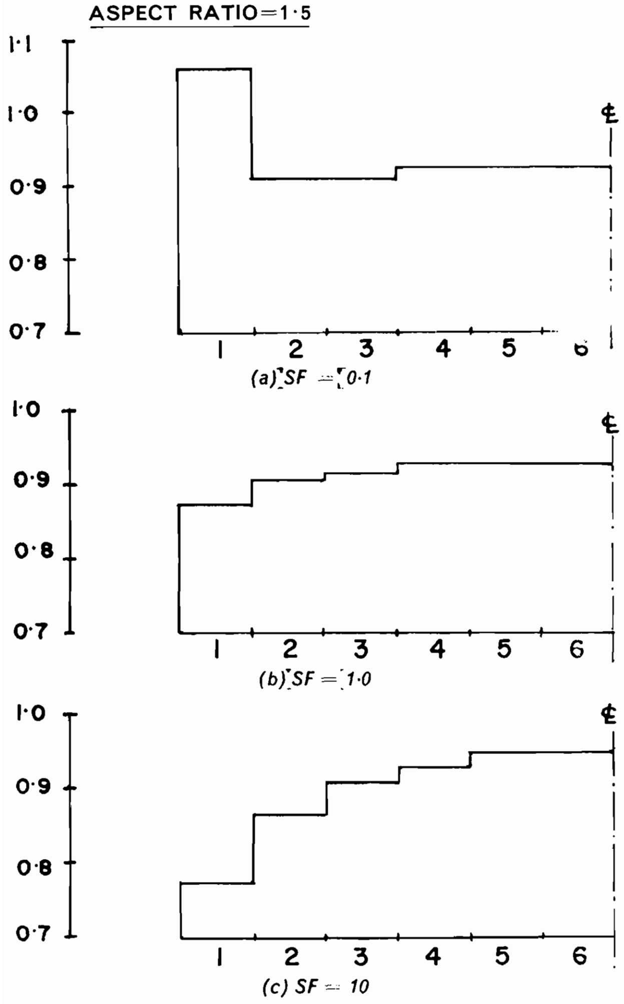


Fig 15. Shear force coefficients in web frame beams



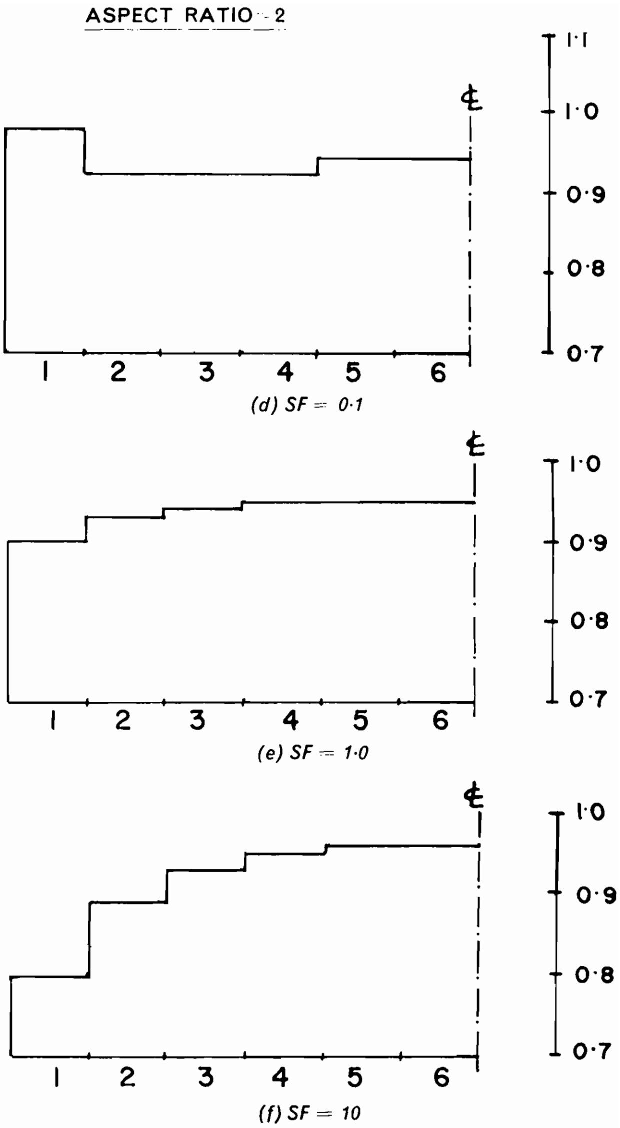


Fig 16. Shear force coefficients in web frame beams

ranges of framed tubes will not only help to save time at the preliminary design stage, but also provide a clearer understanding of the general behaviour of the framed tube systems.

Acknowledgement
The authors gratefully acknowledge the assistance of Mr. Douglas Stoker, structural computer coordinator of Skidmore, Owings & Merrill, during the computer solutions.

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